

Supplementary Information

Synergistic barriers to algorithmic recourse in healthcare and administrative systems

A.C Demidont

Supplementary Table 1: Individual barrier removal effects.

Barrier	Layer	Baseline P (%)	P with removal (%)	Δ (%)	95% CI
Rapid Data Transmission	L1	0.0018	0.0060	0.0042	(0.0038, 0.0046)
Multi-System Integration	L1	0.0018	0.0033	0.0015	(0.0013, 0.0017)
Permanent Storage	L1	0.0018	0.0040	0.0022	(0.0020, 0.0024)
Error Detection Difficulty	L2	0.0018	0.0051	0.0033	(0.0030, 0.0036)
Correction Process Barriers	L2	0.0018	0.0051	0.0033	(0.0030, 0.0036)
Incomplete Correction Propagation	L2	0.0018	0.0045	0.0027	(0.0024, 0.0030)
Awareness Gap	L3	0.0018	0.0060	0.0042	(0.0038, 0.0046)
Record Access Barriers	L3	0.0018	0.0033	0.0015	(0.0013, 0.0017)
Legal Knowledge Gap	L3	0.0018	0.0073	0.0055	(0.0050, 0.0060)
Legal Resource Barriers	L3	0.0018	0.0045	0.0027	(0.0024, 0.0030)
Systemic Bias in Algorithms	L3	0.0018	0.0060	0.0042	(0.0038, 0.0046)

L1 = Data Integration; L2 = Data Accuracy; L3 = Institutional. All individual effects are $<0.02\%$, consistent with Proposition 1. CIs from bootstrap resampling ($n = 1,000$).

Supplementary Table 2: Shapley value attribution.

Barrier	Layer	Shapley value (%)	Relative share	Rank
Legal Knowledge Gap	Institutional	11.3	11.3%	1
Rapid Data Transmission	Data Integration	11.1	11.1%	2
Correction Process Barriers	Data Accuracy	10.7	10.7%	3
Systemic Bias in Algorithms	Institutional	10.5	10.5%	4
Awareness Gap	Institutional	10.0	10.0%	5
Error Detection Difficulty	Data Accuracy	9.6	9.6%	6
Incomplete Correction Propagation	Data Accuracy	8.5	8.5%	7
Permanent Storage	Data Integration	8.3	8.3%	8
Legal Resource Barriers	Institutional	8.1	8.1%	9
Record Access Barriers	Institutional	6.6	6.6%	10
Multi-System Integration	Data Integration	6.3	6.3%	11

Shapley values account for interaction effects across all possible removal orderings, providing fair attribution even under multiplicative blocking.

Supplementary Table 3: Sobol sensitivity indices.

Barrier	S1	S1 95% CI	ST	ST 95% CI	Interaction (ST–S1)
Rapid Data Transmission	0.078	(0.06, 0.10)	0.103	(0.09, 0.11)	0.025
Multi-System Integration	0.079	(0.06, 0.10)	0.103	(0.09, 0.11)	0.024
Permanent Storage	0.077	(0.06, 0.10)	0.103	(0.09, 0.11)	0.026
Error Detection Difficulty	0.078	(0.06, 0.10)	0.104	(0.09, 0.11)	0.026
Correction Process Barriers	0.078	(0.06, 0.10)	0.104	(0.09, 0.11)	0.026
Incompl. Correction Propagation	0.075	(0.06, 0.09)	0.103	(0.09, 0.11)	0.028
Awareness Gap	0.109	(0.08, 0.14)	0.112	(0.10, 0.12)	0.003
Record Access Barriers	0.065	(0.04, 0.09)	0.098	(0.08, 0.11)	0.033
Legal Knowledge Gap	0.110	(0.08, 0.14)	0.111	(0.10, 0.12)	0.001
Legal Resource Barriers	0.075	(0.06, 0.09)	0.103	(0.09, 0.11)	0.028
Systemic Bias in Algorithms	0.078	(0.06, 0.10)	0.110	(0.10, 0.12)	0.032

S1 = first-order (main) effects; ST = total effects including interactions. Saltelli sampling with $n = 1,024$ base samples. ST substantially exceeds S1 for all barriers, confirming interaction dominance.

Supplementary Table 4: Bootstrap robustness summary.

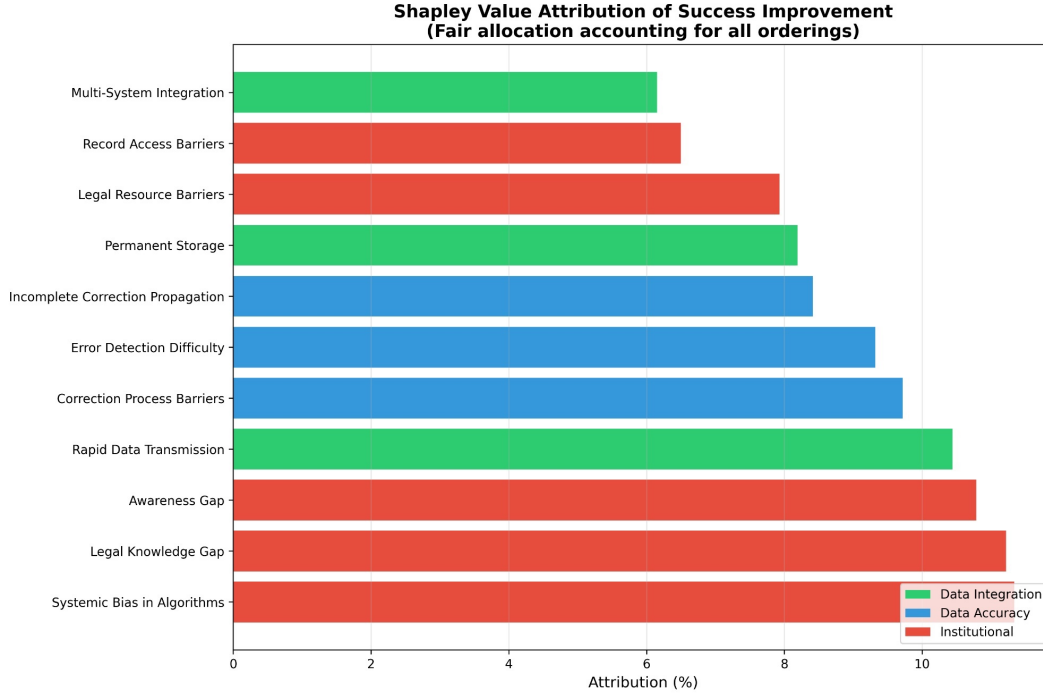
Finding	Threshold	% resamples supporting
Three-way interaction dominance	>70% of variance	100.0%
Maximum individual barrier effect	<1%	100.0%
Barriers needed for 90% success	≥ 10 of 11	100.0%

Bootstrap resampling with $n = 1,000$ resamples and $\pm 10\%$ parameter perturbation.

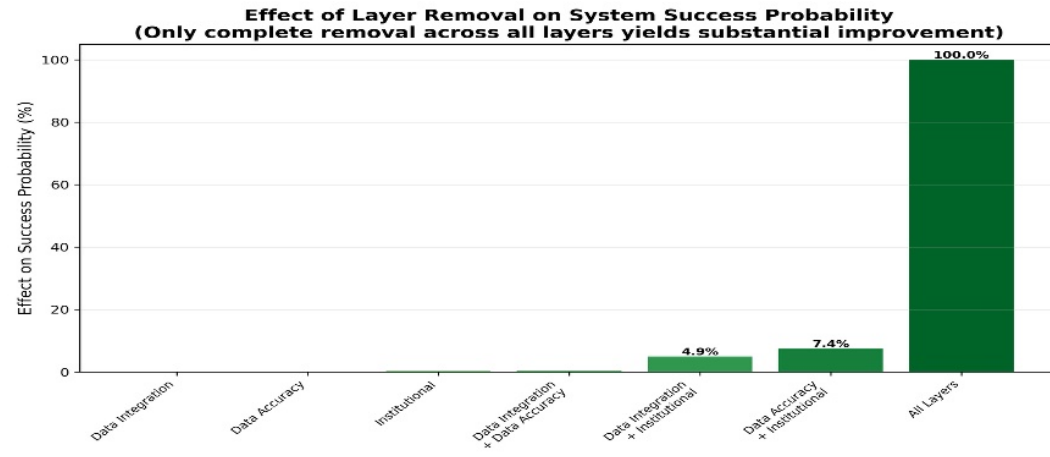
Supplementary Table 5: Signal-to-noise ratio analysis.

Noise level (%)	SNR (dB)	Quality	CV (%)
1	14.7	Good (>10 dB)	5.4
3	11.6	Good (>10 dB)	8.1
5	7.8	Marginal (3–10 dB)	15.2
10	4.7	Marginal (3–10 dB)	33.6
15	2.7	Marginal (3–10 dB)	43.2
20	1.4	Marginal (3–10 dB)	54.1
25	0.4	Poor (<3 dB)	73.2
30	−0.8	Poor (<0 dB)	93.8

SNR remains positive (>0 dB) up to approximately 25% parameter noise. CV = coefficient of variation of output success probability.



Supplementary Figure 1: **Shapley value attribution of barrier contributions.** Shapley value decomposition assigning relative contribution of each barrier to overall system success while accounting for all possible barrier removal orderings. Barriers are coloured by layer: green = Data Integration, blue = Data Accuracy, red = Institutional. Values represent fair attribution of total achievable improvement, with the five highest-contributing barriers spanning all three layers, confirming the cross-layer nature of system dynamics.



Supplementary Figure 2: **Effect of layer removal on system success probability.** Only complete removal across all three layers yields substantial improvement (100%). Single-layer removal produces negligible effects; two-layer combinations yield at most 7.4% (Data Accuracy + Institutional), confirming three-way interaction dominance.