

Supporting Information

Numerical Study on Elastic Properties of Natural Fibres in Multi-Hybrid Composites

Finite Element Analysis

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{Bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{Bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} \quad (S1)$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{Bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & \left(\frac{C_{22} - C_{23}}{2} \right) & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{Bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} \quad (S2)$$

$$E_L = C_{11} - \frac{2C_{12}^2}{C_{22} + C_{23}} \quad (S3)$$

$$E_T = C_{11} - \frac{2C_{12}^2}{C_{22} + C_{23}} \frac{[C_{11}(C_{22} + C_{23}) - 2C_{12}^2]}{(C_{11}C_{22} - C_{12}^2)} (C_{22} - C_{23}) \quad (S4)$$

$$\nu_{in-plane} = \frac{C_{12}}{C_{22} + C_{23}} \quad (S5)$$

$$G_T = C_{66} \quad (S6)$$

Where,

σ : Stress vector

ε : Strain vector

C : Material stiffness matrix

E_L : Longitudinal modulus of elasticity

E_T : Transverse modulus of elasticity

$\nu_{\text{in-plane}}$: In-plane Poisson's ratio

G_T : Transverse shear modulus

Mean Field Homogenisation

$$e_{ij}^c = S_{ijkl} e_{kl}^* \quad (S7)$$

$$S_{ijkl} = S_{jikl} = S_{ijlk} \quad (S8)$$

$$S_{jikl} \neq S_{ijlk} \quad (S9)$$

$$S_{1111} = \frac{1}{2(1-\nu_o)} \left\{ 1 - 2\nu_o + \frac{3\alpha^2 - 1}{\alpha^2 - 1} - \left[1 - 2\nu_o + \frac{3\alpha^2}{\alpha^2 - 1} \right] g \right\} \quad (S10)$$

$$S_{2222} = S_{3333} = \frac{3}{8(1-\nu_o)} \left(\frac{\alpha^2}{\alpha^2 - 1} \right) + \frac{1}{4(1-\nu_o)} \left[1 - 2\nu_o - \frac{9}{4(\alpha^2 - 1)} \right] g \quad (S11)$$

$$S_{2233} = S_{3322} = \frac{1}{4(1-\nu_o)} \left\{ \frac{\alpha^2}{2(\alpha^2 - 1)} - \left[1 - 2\nu_o + \frac{3}{4(\alpha^2 - 1)} \right] g \right\} \quad (S12)$$

$$S_{2211} = S_{3311} = -\frac{1}{2(1-\nu_o)} \left(\frac{\alpha^2}{\alpha^2 - 1} \right) + \frac{1}{4(1-\nu_o)} \left\{ \frac{3\alpha^2}{\alpha^2 - 1} - (1 - 2\nu_o) \right\} g \quad (S13)$$

$$S_{1122} = S_{1133} = -\frac{1}{2(1-\nu_o)} \left[1 - 2\nu_o + \frac{1}{\alpha^2 - 1} \right] + \frac{1}{2(1-\nu_o)} \left[1 - 2\nu_o + \frac{3}{2(\alpha^2 - 1)} \right] g \quad (S14)$$

$$S_{2323} = S_{3232} = \frac{1}{4(1-\nu_o)} \left\{ \frac{\alpha^2}{2(\alpha^2 - 1)} + \left[1 - 2\nu_o - \frac{3}{4(\alpha^2 - 1)} \right] g \right\} \quad (S15)$$

$$S_{1212} = S_{1313} = \frac{1}{4(1-\nu_o)} \left\{ 1 - 2\nu_o - \frac{\alpha^2 + 1}{\alpha^2 - 1} - \frac{1}{2} \left[1 - 2\nu_o - \frac{3(\alpha^2 + 1)}{\alpha^2 - 1} \right] g \right\} \quad (S16)$$

Where,

e : Strain tensor

S : Stiffness tensor

ν_o : Poisson's ratio of the material

g : Scaling factor

α : Aspect ratio of the fibre length to the fibre diameter

Rule of Mixture

$$E_L = E_f V_f + E_m V_m \quad (S17)$$

$$v_{in-plane} = v_f V_f + v_m V_m \quad (S18)$$

$$E_T = \frac{E_m E_f}{E_m V_f + E_f V_m} \quad (S19)$$

$$G_T = \frac{G_m G_f}{G_m V_f + G_f V_m} \quad (S20)$$

Where,

E_f : Modulus of elasticity of the fibre

E_m : Modulus of elasticity of the matrix

V_f : Volume fraction of the fibre in the composite

V_m : Volume fraction of the matrix in the composite

Chamis Model

$$E_L = E_f V_f + E_m V_m \quad (S21)$$

$$v_{in-plane} = v_f V_f + v_m V_m \quad (S22)$$

$$E_T = \frac{E_m}{1 - \left\{ \sqrt{V_f} \left[1 - \left(\frac{E_m}{E_f} \right) \right] \right\}} \quad (S23)$$

$$G_T = \frac{G_m}{1 - \left\{ \sqrt{V_f} \left[1 - \left(\frac{G_m}{G_f} \right) \right] \right\}} \quad (S24)$$

Where,

G_m : Shear modulus of the matrix material

G_f : Shear modulus of the fiber material

Composite Cylinder Assemblage

$$E_L = E_f V_f + E_m V_m - \frac{2 E_m E_f V_f (v_f - v_m)^2 (1 - V_f)}{E_f B_1 + E_m B_2} \quad (S25)$$

$$v_{in-plane} = v_f V_f + v_m V_m + \frac{(V_f V_m (v_f - v_m) (2 E_f v_m + v_m E_f - E_f + E_m - E_m v_f - E_m v_f))}{(E_f B_1 + E_m B_2)} \quad (S26)$$

Where

$$B_1 = 2\nu_m \mathcal{V}_f - \nu_m + \nu_m V_f - 1 - V_f$$

$$B_2 = 2\nu_f - V_f \mathcal{V}_f - V_f \mathcal{V}_f + V_f + \nu_f - 1$$

$$E_T = 2(1 + \nu_{out-plane}) G_T \quad (S27)$$

$$\nu_{out-plane} = \frac{(K' - m G_T)}{(K' + m G_T)} \quad (S28)$$

Where

$$m = 1 + 4K' \left(\frac{\nu_{in-plane}^2}{E_L} \right) \quad (S29)$$

$$K' = \frac{K_m (K_f + G_m) V_m + K_f (K_m + G_m) V_f}{(K_f + G_m) V_m + (K_m + G_m) V_f} \quad (S30)$$

$$K_f = \frac{E_f}{2(1 + \nu_f)(1 - 2\nu_f)} \quad (S31)$$

$$K_m = \frac{E_m}{2(1 + \nu_m)(1 - 2\nu_m)} \quad (S32)$$

$$A \left(\frac{G_T}{G_{matrix}} \right) + 2B \left(\frac{G_T}{G_{matrix}} \right) + C = 0 \quad (S33)$$

$$A = 3V_f(1 - V_f) \left(\frac{G_f}{G_m} - \right) \left(\frac{G_f}{G_m} + \eta_{fiber} \right) + \left[\frac{G_f}{G_m} \eta_m + \eta_f \eta_m - \left(\frac{G_f}{G_m} \eta_m - \eta_f \right) V_f \right] \times \left[V_f \eta_m \left(\frac{G_f}{G_m} - \right) - \left(\frac{G_f}{G_m} \eta_m + 1 \right) \right] \quad (S34)$$

$$B = -3V_f(1 - V_f) \left(\frac{G_f}{G_m} - \right) \left(\frac{G_f}{G_m} + \eta_f \right) + \left(\frac{1}{2} \right) \left[\eta_m \frac{G_f}{G_m} + \left(\frac{G_f}{G_m} - 1 \right) (V_f + 1) \right] \times \left[(\eta_m - 1) \left(\frac{G_f}{G_m} + \eta_f \right) - 2 \left(\frac{G_f}{G_m} \eta_m - \eta_f \right) V_f \right] \quad (S35)$$

$$+ \left[\frac{V_f}{2} (\eta_m + 1) \left(\frac{G_f}{G_m} - 1 \right) \right] \times \left[\left(\frac{G_f}{G_m} + \eta_f \right) + \left(\frac{G_f}{G_m} \eta_m - \eta_f \right) V_f \right]$$

$$C = 3V_f(1 - V_f) \left(\frac{G_f}{G_m} - \right) \left(\frac{G_f}{G_m} + \eta_f \right) + \left[\eta_m \frac{G_f}{G_m} + \left(\frac{G_f}{G_m} - 1 \right) V_f + 1 \right] \times \left[\left(\frac{G_f}{G_m} + \eta_f \right) + \left(\frac{G_f}{G_m} \eta_m - \eta_f \right) V_f \right] \quad (S36)$$

Where,

$\nu_{out-plane}$: out-of-plane Poisson's ratio

m : scaling factor

K : stiffness parameter

Composite Density

$$\rho_c = \rho_f V_f + \rho_m V_m \quad (S37)$$

Where,

ρ_c : Density of composite

ρ_f : Density of the fibre

ρ_m : Density of the matrix

V_f : Volume fraction of the fibre in the composite

V_m : Volume fraction of the matrix in the composite

Comparative Study

$$A_E = \frac{E_L}{E_T} \quad (S38)$$

$$\eta E_L = \frac{E_{L(comp)} - E_m V_m}{E_f V_f} \quad (S39)$$

$$HEI = \frac{E_{L(hyb)} - \sum w_i E_{L(i)}}{\sum w_i E_{L(i)}} \times 100\% \quad (S40)$$

$$Specific\ stiffness = \frac{E_L}{\rho} \quad (S41)$$

Where,

A_E : Anisotropy ratio of the composite

ηE_L : Reinforcement efficiency of the composite

HEI : Hybrid effect index of the composite

w_i : Relative volume fraction of each fibre in the composite